Hydroextraction: Flow in Submerged Cakes

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Flow equations have been proposed for hydroextractor cakes wherein the pores are filled with moving liquid. These equations have been confirmed for the effect of each variable by use of different experimental techniques. The critical wetting rate has provided the most informative steady state, in which the cake is fed with liquid to maintain the inner surface of the liquid at radius r_L coincident with that of the cake at radius r_c. Transient techniques have also been used either with a probe to time the movement of a liquid level between two radii less than r_c, or with a photoelectric detector to observe the coincidence of $r_L = r_c$. The tests have proved the validity of the hydrodynamic assumptions involved in the equations and have provided both precise methods for research and practical methods for industrial purposes.

Analysis of the batch operation of centrifugal hydroextractors is complicated by the succession of hydrodynamic conditions which occur as a cake is built up at low speed, spun at high speed to remove liquor, washed, and spun again to remove adhering solvent. During the cake-building period it may well be that the pores run full of liquid, whereas in the spinning periods it is probable that the majority of cake interstices contain air, the liquid flowing over the bounding surfaces of the cavities. At least two hydrodynamic conditions thus require analysis. The operation of truly continuous hydroextractors should then respond to the application of the relationships derived from these analyses. The batch hydroextractor is governed by relations incorporating the same flow principles, where they are appropriate in the cycle of operation, but requires the solution of the transient equations for the varying centrifugal force during acceleration and deceleration periods of the cycle.

A possible cycle is outlined in Table 1, the pumping force f moving the liquid increasing with increasing rotational speed and the hydrodynamic condition depending on whether the pores run full of liquid F or contain air O. The subdivisions shown are only approximations of the true system in certain periods but they suffice to show the necessity for analyzing specific cases involving selected hydrodynamic conditions, which can then be integrated to provide the whole cycle. The search for the best operating conditions in a particular hydroextraction problem can then be attempted on the basis of the known relationships governing all parts of the cycle. The essential mechanisms requiring understanding are the steady states governing the flow of liquid through (*a*) full pores, e.g., at a moment in operation 1; and (*b*) across the surfaces around open pores, e.g., at a moment in operation 3. These can then be modified for the time functions to cope with movement of the free liquid surface within the cake and with changing centrifugal forces. It is simplest first to study a steady state through which the actual operation passes. A most valuable condition is that of the "critical wetting rate," at which a liquid surface is maintained just at the inner surface of the cake for given cake thickness and hydroextractor speed.

Singularly little endeavor has been applied to the study of the fundamental mechanism of the hydroextraction process, despite its importance in chemical engineering. The researches on the subject conducted by the author and associates concentrated initially (*1, 5, 6, 21*) on problems in which the fundamental mechanism is the flow of liquid through packed beds within a centrifugal field, the interstices between the cake particles being filled with liquid. The flow relationships derived from theoretical considerations were tested for each variable and found satisfactory. They were then applied successfully to the comparison of centrifugation and filtration in both hydroextractor cakes and incompressible porous structures of similar geometrical form. These tests showed that there was no idiosyncrasy in flow in a centrifugal field within the submerged cake. The present review shows how the first fundamental mechanism has been analyzed and applied to the study of cake formation. The relationships can now be applied to the f_1, r_1 problems directly and to the f_1^*, r_1^* problems by appropriate modification for the transient variables with time.

The second major mechanism is concerned with flow through open pores. A report is to be published of work (*18*) aimed at the problems associated with liquid movement and retention within cakes when the void space is not filled with liquid. Knowledge of these two groups of fundamental problems concerning f_2, r_2 and f_2^*, r_2 should allow of considerable progress in the analysis and prediction of the course of complete hydroextraction cycles.

THEORETICAL BASES

The equations suitable for expressing the flow rates are founded on a mechanism requiring a number of limiting assumptions.

1. The effect of the gravitational field is negligible compared with that of the centrifugal field; therefore the liquid always flows through the cake in a plane normal to the axis of rotation of the basket.
2. The interstices at all points in the cake are filled with liquid when $r_L \leq r_c$ (Figure 1, ref. *7*) and the liquid moves in streamline flow through the cake. The latter is probably true for cake materials giving permeabilities of the order of 10^{-7} g./sec.[2]
3. The liquid flows (radially) outward from the hydroextractor basket, tangential movement being due only to liquid filling the expanding cross section to flow in moving outward.
4. Kinetic energy changes due to changes of the radial component of the space velocity may be neglected, the only significant energy requirement being that to overcome frictional resistance to the liquid movement within the cake.
5. The permeability of the cake may be considered uniform in a preliminary analysis of the system. As will be shown later, it may be necessary to assess permeability distributions for precise analysis in most systems in order to cope with compression or size distributions in cakes and with cloth-cake interface resistance.
6. The highly permeable filter cloth between the cake and the basket wall does not run full of liquid, which means that atmospheric pressure is exerted on the liquid at the outer radius of the cake.

These assumptions were used in the following derivations and the experimental work was designed to test all the variables independently as far as possible. This requirement led to the development of many methods of measuring the effect of certain variables for elaborate cross-checking rather than for industrial use, though certain techniques have proved preferable and can be recommended for specific purposes such as precise research or the testing of operational hydroextractors. It was considered necessary to obtain both the correlation of hydroextractor data with theoretical equations representing the mechanism and an agreement between centrifugation and filtration data. If the hydroextraction equations are sound, they must contain a permeability K_C which is very similar to the filtration permeability K_F. The probability is that they should be the same, but in some cases secondary

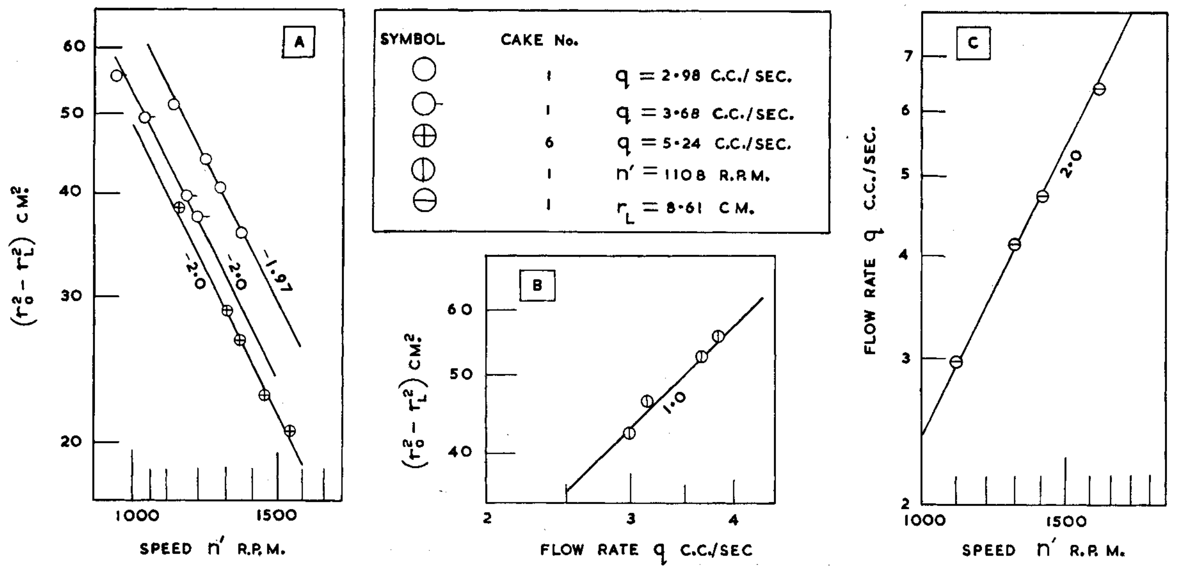

Fig. 1. Tests based on static r_L measurements (Method I), by use of a No. 1 cake of precipitated chalk of thickness 1.55 cm. and a No. 6 cake of cornstarch of thickness 0.98 cm. (*7*).

effects such as the Coriolis acceleration might lead to a flow mechanism significantly different from that in normal filtration (*20*). Apart from such unusual effects, the secondary intention of the work was thus to prove agreement between K_C and K_F to the 1% level. It was considered that once checked with such precision the flow equations could form a sound basis for developed relationships such as the relationship for cake formation. The simple equations developed do not include any effect of compressive stress on cake permeability. Though following the deviation of K_C from K_F in early work extensive studies were made to test the effect of compressive stress in filtration cells and hydroextractor basket (*5*), similar to the studies of Grace (*3*, *4*), the present opinion is that compressive effects have been of minor importance in the work on kieselguhr, metal dust, cornstarch, chalk, and barium sulfate. The early results suggesting compressive effects have been found due to misapplication of critical wetting techniques and lack of attention to cake history. If cake materials are found which show effects of compression on permeability, analytical methods similar to those of Grace should prove valuable.

FLOW EQUATIONS

The fundamental equation providing the key for all developed forms is the steady state equation for flow through the cake when the positions of the liquid surface r_L and the cake thickness $(r_0 - r_c)$ are invariant with time. With relatively low-permeability cakes and thus with relatively slow flow-rate changes, the developed forms such as for cake formation can be found by neglecting acceleration terms and using the steady state equation as the differential form for integration over a range of r_L or r_c.

The assumptions above lead to an energy balance between the pressure development due to the centrifugal field [Equation (1)] and the energy dissipation by friction over an elemental annulus of radius r and thickness δ_r within the cake [Equation (2)] if the liquid is assumed in steady flow without significant energy terms owing to radial acceleration.

$$\delta P_c = \frac{(2\pi n)^2}{g} \cdot r \cdot \delta r \tag{1}$$

$$\delta P_F = \frac{1}{K_c} \cdot \frac{\mu g}{2\pi X} \cdot \frac{1}{r} \cdot \delta r \tag{2}$$

The full-pressure differences across the system are $(P_c)_0$ developed from r_L to r_0 and $(P_F)_0$ required to maintain flow from r_c to r_0.

$$(P_c)_0 = \frac{1}{2} \cdot \frac{(2\pi n)^2}{g} \cdot (r_0^2 - r_L^2) \tag{3}$$

$$(P_F)_0 = \frac{1}{K_c} \cdot \frac{\mu g}{2\pi X} \cdot \ln \frac{r_0}{r_c} \tag{4}$$

For a balanced system, the values must be equal, $(P_c)_0 = (P_F)_0$, to maintain atmospheric pressure as datum level at the liquid faces at r_L and r_0. Throughout the cake there will be a pressure of $(P_c - P_F)$ above atmospheric pressure as datum at the radius r within the cake, $r_c < r < r_0$, depending on the relationships between the groups in Equations (3) and (4) with r in place of r_0. This is discussed later in terms of the tests on banded cakes.

The steady state leads to the flow-rate equation from the equality $(P_c)_0 = (P_F)_0$ when r_L is not greater than r_c, i.e., with the liquid filling all the cake voids.

$$q = \frac{4\pi^3 n^2 K_c X}{\mu g} \cdot \frac{(r_0^2 - r_L^2)}{\ln r_0/r_c} \tag{5}$$

A useful experimental technique involves the detection of the critical wetting rate when $r_L = r_c$, the liquid surface coinciding with the inner face of the cake,

$$q_w = \frac{4\pi^3 n_c^2 K_c X}{\mu g} \cdot \frac{(r_0^2 - r_c^2)}{\ln r_0/r_c} \tag{6}$$

$$= \frac{4\pi^3 n_c^2 K_c X}{\mu g} \cdot (r_0 + r_c) \cdot r_m \tag{6a}$$

where r_m is the logarithmic mean of r_0 and r_c. The verification of the postulated flow mechanism and related assumptions could be provided by testing Equations (5) and (6) with r_L independent of time, or r_L could be allowed to vary and the resulting time functions applied to the analysis of the experimental data. Both methods were used in order to test the fundamental equations in every conceivable way. Nearly all the tests were carried out on a 9-in.-diam., 4½-in.-depth hydroextractor with a duck-weave cloth behind the cake, though the effect of r_0 was checked by experiments on an 18-in.-diam. hydroextractor.

ANALYTICAL METHODS AND EXPERIMENTAL TECHNIQUES

Method I, Based on r_L Measurements When r_L Is Independent of Time

For the use of Equation (6) in analyzing data, the experimental results must provide flow rate q_w at which $r_c = r_L$ for various speeds n. The early technique (*1*, *2*, *5*), probably similar to that of Smith (*19*), using visual observation of the critical wetting rate, proved dangerously deceptive, and a far superior technique is the use of a photoelectric diffuse-reflection detector (*16*). The light from a lamp L is reflected back to the photocell C (Figure 2, ref. *15*) in increasing intensity as the liquid thickness $(r_c - r_L)$ is reduced. Thus if a cake revolving at a speed n is fed with a constant liquor rate q so that $r_L < r_c$, the photocell output will represent the

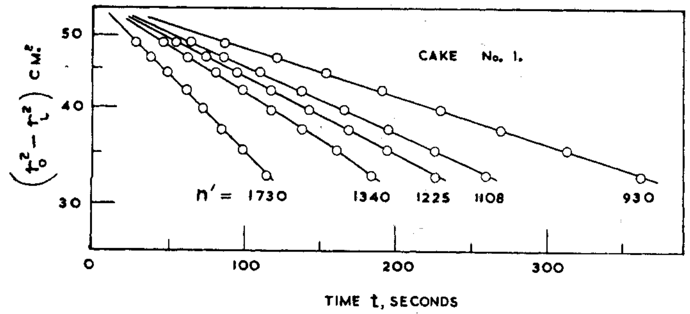

Fig. 2. Tests based on measurement of r_L variations with time (Method II), by use of a No. 1 cake of precipitated chalk (7).

steady state. If n is increased to a new steady value, r_L will change to a new steady state at a larger value of r_L. These positions may be plotted as a graph of photocell output against n. When the liquid film disappears into the cake surface the cell output, a curve nearly linear in n, changes abruptly to a value nearly independent of n. The break point gives a sensitive test of the critical speed n_c at which $q = q_w$ is the critical wetting rate for the cake. This technique is valuable in industrial practice, where the more elaborate probe technique may not be available outside the research laboratory. The logarithmic plot of q_w against n_c for a given cake gives a clear confirmation of the power 2.0 for revolution speed. There are experimental limitations which must be carefully observed in all hydroextractor tests. The cake must be formed under liquor and maintained under liquor throughout all tests. It must first be run at the highest test speed for a short time. Thereafter the results will be reproducible on a q_w, n_c line of gradient 2.0 on a logarithmic plot for all values of n_c (*16*).

A valuable research technique of high precision is provided by the probe indicator developed by Haruni (*6*). For any steady state q and n, the position of r_L could easily be measured to 0.01 cm., the probe I being moved vertically and radially by calibrated hand wheels (D and M in Figure 1, ref. *7*). With this apparatus the variables in Equation (5) could be examined in various ways for a specific cake and liquid, i.e., for constant r_0, r_c, X, μ, K_c, and for $r_L < r_c$.

From Equation (5) for the steady state flow, for constant r_L, q should vary with n^2. For constant n, the variation of r_L being measured with q, each r_L, q pair being for a steady state, the plot of log $(r_0^2 - r_L^2)$ against log q should be a straight line of gradient 1.0. For constant q, corresponding r_L, n pairs being measured, a plot of log $(r_0^2 - r_L^2)$ against log n should be a straight line of gradient -2.0. The agreement with Equation (5) is shown by the data (*7*) on Figure 1 and corroborates the form of the equation and the substantial constancy of r_c and K_c, notably with varying compressive stress for the test material.

Method II, from Drainage Times Based on r_L Changing with Time

The technique consists in stopping the feed of liquor to the basket when r_{L_0} is considerably less than r_c and following the changes of r_L with time. Using the probe, the time t is found for r_L to increase from r_{L_0} at $t = 0$ to r_L at $t = t$. If it is assumed that the steady state equation (5) can be used as the differential equation for the rate of change of V_L, the volume of liquor held between r_L and r_c, the variation of r_L with t is given by

$$q = -\frac{dV_L}{dt} = 2\pi X r_L \cdot \frac{dr_L}{dt} \qquad (7)$$

whence with Equation (5)

$$\ln\left[\frac{r_0^2 - r_{L0}^2}{r_0^2 - r_L^2}\right] = \frac{4\pi^2 n^2 K_c}{\mu g \log r_0/r_c}\cdot t \qquad (8)$$

For a selected value of r_{L_0}, a series of tests of r_L against t with constant μ, n, K_c, r_0, r_c should give a straight line of gradient -1 for the plot of log $(r_0^2 - r_L^2)$ against t. Figure 2 shows the agreement with this form for a cake for various hydroextractor speeds.

Method III, Based on Drainage Times for a Specific Volume of Liquid

With the basket running at a constant speed n, with $r_L = r_c$, the feed can be stopped and simultaneously the volume $V_L{}^1$ added to the basket. If this volume of liquid is assumed to accelerate to the appropriate basket speed immediately, functions can be derived from Equation (5) to show the time required for the liquid surface to attain a selected position in terms of the cake volume V_c and the volume of liquid held within the cake radius r_c.

From Equation (5)

$$q = -\frac{dV_L}{dt} = \frac{4\pi^2 n^2 K_c}{\mu g}\cdot\frac{(V_L + V_c)}{\ln r_0/r_c} \qquad (9)$$

$$= \gamma(V_L + V_c) \qquad (10)$$

where γ is a constant for given cake, liquor, and speed. One method consisted of an analysis of the variation of flow rate with liquid volume V_L, which should give for constant γ a straight line for q against V_L with an intercept $q = \gamma \cdot V_c$, when $V_L = 0$ and $r_L = r_c$. The experimental data are compared (*9*) with a lattice of straight lines of parameter γ, each line having its gradient γ linked with its intercept $\gamma \cdot V_c$ at $V_L = 0$. These rate data are less readily obtained than the time t for the liquid volume to attain V_L from an initial volume $V_L{}^1$ when $t = 0$. In this case the volume $V_L{}^1$ would be added to a critically wetted cake with $r_L = r_c$ and time t found for r_L to attain a selected position,

$$t = \frac{1}{\gamma}\cdot\ln\left[\frac{V_c + V_L{}^1}{V_c + V_L}\right] \qquad (11)$$

or if liquid is allowed to drain to $r_L = r_c$ and $V_L = 0$

$$t^1 = \frac{1}{\gamma}\cdot\ln\left[\frac{V_c + V_L{}^1}{V_c}\right] \qquad (12)$$

An example of such a data series for various $V_L{}^1$ for constant γ and V_c is shown in Figure 3 and also in Figures 2, 3, and 5 of ref. *15*. The critical wetting condition can be found by using the photoelectric detector, the output of which will show a clear break point when $r_L = r_c$. Figure 2 (*15*) shows a photocell fitted to a porous ceramic cylinder used as an incompressible hydroextractor cake; Figure 3 (*15*) shows a series of critical wetting tests at constant n for various added volumes $V_L{}^1$; the break-point data transferred to Figure 5 (*15*) indicate the straight-line relationship between ln $(V_c + V_L{}^1)$ and t^1 from Equation (12) with an intercept at ln V_c when $t^1 = 0$. The change of gradient γ with speed n is clear from Figure 5 of ref. *15* and on Figure 4 the data confirm the expected relationship $\gamma \propto n^2$. Even with visual observation, this method can be used to obtain γ and hence K_c for selected conditions on operating plant. The assumption of negligible time for the liquid $V_L{}^1$ to be accelerated to the basket speed is sound for the materials used here but would lead to curvature in these plots for high-permeability cakes giving low t^1 values of the same order as the acceleration time required.

HYDROEXTRACTION AND FILTRATION PERMEABILITIES

The methods described above have been used for testing the fundamental equation in detail (*7*) and show good agreement for a given cake and liquor. These techniques involve the variation of r_L, q, n, but not the variation of r_0 or μ. By use of centrifuges of different r_0 and liquids of various viscosities, the effect of these variables has been shown to fit Equation (5) and its derived forms. The dependance on the first power of μ confirms the assumption of streamline flow and the negligible effect of radial acceleration kinetic energy terms (*7*).

The success of these methods means that the assumptions and derived equations are probably correct in that they lead to a complete correlation of hydroextraction data in terms of a permeability K_c for a given cake, the same value being obtained by various methods. The effect of compressive stress appears negligible with the materials used. The proof of the correctness of the equations still lacks agreement between K_c and K_F. Much experimental work has been done on this, leading to detailed analyses of many forms of experiment specifically designed to test this agreement to 1% accuracy (*8* to *15*). The comparison of K_c in a 9-in.-diam. hydroextractor with K_F from samples cut from the cake proved unsatisfactory (*2*, *5*, *6*).

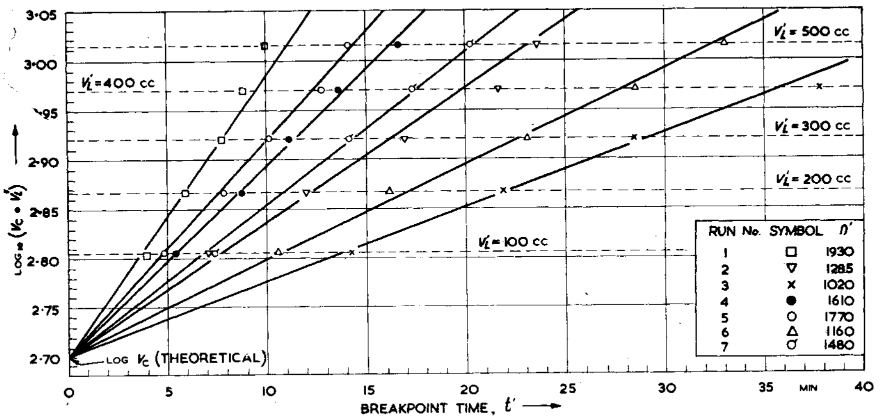

Fig. 3. Break-point data for drainage tests in ceramic cylinders. Cylinder II was of maximum pore size, 2μ and thickness 0.79 cm. (*15*).

In a cylindrical cell of 1½-in. diam. small cakes of thickness up to about 1½ in. were made which could be tested in both the centrifugal field and the gravitational field (*8*), both K_c and K_F being obtained from the cake without disturbing it. The cell (shown diagrammatically in Figure 8, ref. *8*) was held in a carrier within the hydroextractor and whirled about the center line, a liquid level being maintained at G, while the liquor discharged freely into the atmosphere at the perforated base of the cell. The cell was filled completely with liquid. The cake B held on cloth C had to be protected during formation and testing (*8*) owing to the strong secondary circulation in the space A. For filtration tests the cell was connected to a head tube and K_F obtained from the time for the liquid level to fall from H_1 to H_2:

$$t = \frac{a\mu(r_0 - r_c)}{K_F A} \cdot \ln \frac{H_1}{H_2} \qquad (13)$$

where A is the cross-sectional area of the cake face, and the thickness of the cake is $(r_0 - r_c)$.

In this cell the hydroextraction permeability K_c must be found from a relation similar to Equation (5) but appropriate to the constant cross-sectional area A of the cell.

$$q = \frac{2\pi^2 n^2 A K_c}{\mu g} \cdot \frac{r_0^2}{(r_0 - r_c)} \qquad (14)$$

Tests on the small cell were under centrifugal and gravitational pressure heads similar to those in the large hydroextractor, and the results showed disagreement of 5 to 20% between K_c and K_F. Many tests with this cell and with samples from the hydroextractor cakes of various materials (*8*) showed that though the proof was nearly complete, it could not be attained with the complex experimental conditions and possible cake changes with fine powders for bed material. Tests on incompressible porous ceramic cylinders of dimensions near those of the hydroextractor, of which the figures mentioned above are examples, showed reproducible agreement between K_c and K_F to 5% (*15*) but observable local variations in the permeability of such material probably accounted for the remaining difference. Tests on small sintered metal plugs (Figure 9, ref. *15*) comparable to the small cell work showed the consistent agreement to 1% (*15*), which is regarded as the final proof that the functions proposed are probably correct for the flow mechanism in submerged cakes of incompressible materials.

PRESSURE AND PERMEABILITY DISTRIBUTIONS IN CAKES

Though the agreement between K_c and K_F from the average permeability functions described above was near enough to show that the proposed mechanism was probably correct, the discrepancies were consistent, and in seeking their cause and greater precision it has been found that significant distributions of permeability may occur within the cake. Analyses of cakes of various thicknesses can be explained reasonably on the basis of relatively small changes in K_c within the cake and a significant resistance at the cloth-cake interface (*10*). This seems clear from the examples shown in Table 2 for cakes of various thicknesses.

The layer permeabilities are found from the analysis of the complete cake permeability K_m in terms of the layer values $K_1, K_2, \cdots K_n$, the inner radius of the n-th layer r_n being r_c, the inner radius for the cake.

$$\tfrac{1}{2}(2\pi n)^2(r_0^2 - r_L^2) = \frac{\mu q g}{2\pi X K_m} \cdot \ln \frac{r_0}{r_c}$$
$$= \frac{\mu q g}{2\pi X}\left[\frac{\ln r_0/r_1}{K_1} + \frac{\ln r_1/r_2}{K_2} + \cdots + \frac{\ln r_{n-1}/r_n}{K_n}\right] \qquad (15)$$

The results show a high resistance in the layer next to the cloth, which can be ascribed to the cloth-cake interface. The cloth tested on its own, when in a dirty condition after use, showed only a flow resistance of about 1% of the cakes.

An interesting study allied to the permeability distribution work is an assessment of the fundamental pressure distribution. The balanced operation in steady state flow requires $(P_c)_0$ to be equal to $(P_F)_0$. Thus it is clear that there are likely to be positions in the cake where the pressure on the liquid is less than atmospheric. The maximum vacuum is probably equivalent to about 100 cm. of water and it is not likely to produce degassing of the liquid, which might cause discontinuities in the pressure distribution along the radial direction. However, it is necessary to test the pressure distribution indirectly so far as possible. By making cakes from layers of differing permeabilities, it is possible (*11*) to test whether the resultant flow rates fit those predicted from the known permeabilities and the assumed pressure distribution of $(P_c - P_F)$.

From detailed analyses of tests with $r_L < r_c$ on various cake thicknesses of a single material in the hydroextractor and in the small centrifuge cell it is possible to assign a value for permeability for any layer of selected material at a particular position when compressed by a particular thickness of material nearer the axis of rotation (*10*, *11*). Figure 5 exemplifies the assessment of such cases in the hydroextractor. The cakes were made from layers of cornstarch and kieselguhr of differing permeabilities (Table 2). The dimensions, permeabilities, and flow rates for specific speeds were known, and thus the P_c distribution is known, rising from $P_c = 0$ at $r_L = 8$ cm. to a maximum $(P_c)_0$ at $r_0 = 10.8$ cm. along the curve *Aah*. The P_F curve can be calculated as rising almost proportionally to the radial thickness of the cake from zero at r_c to the total value for the particular cake layer; e.g., the P_F requirement for the innermost layer of starch in cake No. 8 is given by *ab*. The net value of $(P_c - P_F)$ at the inner radius of the kieselguhr layer is given by the ordinate at *b*. The same flow rate leads to a pressure drop for the outermost starch layer of *fg*. The equivalent development of P_c through this layer would be *ef*. Thus excess pressure at the outer layer of the kieselguhr is given by the ordinate at *e*, when *g* is fixed at the atmospheric pressure datum for the outlet at r_c. If the kieselguhr layer runs full, the P_c development is from *b* to *c*. Balanced operation would result if the pressure drop due to friction through the kieselguhr layer is *ce*. Actually in this example it would be *de*. The discrepancy *cd* can be explained only by the assumption that the kieselguhr layer did not develop the centrifugal pressure rise *bc* but only *bd*. This means that this layer was not running full of liquid, and it has been confirmed from many tests that this seems the true explanation, a constant pressure region occurring over part of the layer thickness. The case of cake No. 4 with the kieselguhr as the inside layer gives a good agreement. From the experimental *q* and known *K* values, the estimated and actual pressure curves comparable to *bc* and *bd* are very close together, an indication that the kieselguhr layer runs full. Cake No. 3 exemplifies cases in which a high-permeability layer on the outside will not run full. The *q*, *K* data for the inner starch layer running full gives a value of $P_c - P_F$ of zero at the inner layer of the kieselguhr, which means that the atmospheric pressure obtains at the outer radius of the starch layer, the high permeability layer running empty. This type of data from many different cases (*11*) is considered to prove that layers of high permeabilities within a cake will not run full and thus will not develop their maximum centrifugal-pressure contribution and that such layers will not run full on the outside of a cake. This latter case justifies the assumption that the highly permeable cloth layer in a centrifuge cake does not run full and thus does not enter into the resistance functions leading to the flow equations. The conditions demonstrated as leading to these discontinuities in pressure development are absent in uniform cakes of a single material, which affords some indirect proof that the normal cake runs full as originally assumed. It seems unlikely from these tests that the vacuum developed in normal cakes would cause discontinuity in the liquid path sufficient to affect the continuous pressure development according to Equation (1).

TABLE 1

OPERATION SEQUENCE IN BATCH HYDROEXTRACTOR

f = centrifugal pumping force
R = flow resistance

	Operation	Speed	Pore flow Initial	Pore flow Final	Hydrodynamic variables
1	Cake formation	Low	*F*	*F*	f_1^*, R_1^*
2*a*	Cake drainage	Increasing	*F*	*F*	f_1^*, R_1
2*b*		High	*F*	*O*	f_1, R_1 to f_2, R_2
2*c*		Decreasing	*O*	*O*	f_2^*, R_2
3	Cake washing	Low	*O* or *F*	*O* or *F*	f_1, R_1 or f_2, R_2
4*a*	Wash water drainage	Increasing	*O* or *F*	*O*	f_1^*, R_1 to f_2^*, R_2
4*b*		High	*O*	*O*	f_2, R_2
4*c*		Decreasing	*O*	*O*	f_2^*, R_2

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*The major variable is altering during the period; e.g., as the cake forms, the pumping force f_1 will increase as the liquid bulk in the basket increases and the flow resistance r_1 will increase with cake thickness.

EFFECT OF CAKE SHAPE

The formation of cakes differing from the assumed form with constant r_0 and r_c at all positions round the basket and along the vertical height will produce two effects: first, the obvious complication in deducing a flow function leading to the cake-formation function and, second, the problem of proper derivation of the steady state flow equation comparable to Equation (5). The latter problem has been assessed (*12*) by estimating the discrepancy between the flow through a wedge-shaped cake from an Equation (5) based on a mean value of r_c and the true flow from a relaxation solution to the Laplace equations for filtration and centrifugation. These analyses show that in general the simple analysis based on Equation (5) and the mean cake thickness will be satisfactory for most purposes, even with quite large deviations from the theoretical cake form.

The second problem of cake formation has been assessed (*14*) in a manner different from that suggested by Maloney (*17*). It is clear that the method of feeding the slurry to the basket will be all-important. Cake formation by growth all over its inner face is not ensured by the cake being formed underneath a liquor layer or proved by the fact that a cylindrical cake, i.e., one with a vertical inner face, is produced finally. The data in Figures 1 to 5 of reference *14* show the very different type of growth found with slurry fed to the bottom of the basket with a liquor level maintained at a fixed r_L.

The uniform growth of the cake of permeability K_c at all facial positions starting from a layer of inner radius r_0, outer radius r_w, and permeability K_0 would be given by an appropriate integration of the steady state flow-rate equation:

$$q = \frac{\dfrac{4\pi^3 n^2 X(r_w^2 - r_L^2)}{\mu g}}{\left[\dfrac{\ln r_0/r_c}{K_c} + \dfrac{\ln r_w/r_0}{K_0}\right]} \tag{16}$$

Shortly after the growth starting the K_0 layer ceases to run full if its permeability K_0 is, say, ten times K_c, the term in K_0 becomes inoperative and $(r_w{}^2 - r_L{}^2)$ becomes $(r_0{}^2 - r_L{}^2)$. For a cake growing on an existing cylindrical layer of the same material K_0 becomes K_c and r_0 disappears from the final bracketed term. This equation is valid for cakes of materials such as starch, chalk, kieselguhr, and barium sulfate, where the sedimentation rate is so high that the liquid layer from r_L to r_0 is effectively clear liquid rather than slurry.

For a slurry volume V added from the start of cake formation

$$r_c{}^2 = r_0{}^2 - \frac{sV}{\pi X \sigma_p} = r_0{}^2 - C_1 V \tag{17}$$

and the relationship between slurry feed rate q_s and liquid drainage rate q for a fixed r_L is

$$q = q_s\left(1 - \frac{s}{\sigma}\right) \tag{18}$$

whence

TABLE 2

PERMEABILITY OF LAYERS IN 9-IN. HYDROEXTRACTOR.

Starch

Layer number*	1	2	3	4	5	6	7	8	9
W†, g.	300	400	500	600	700	800	900	1,000	1,100
Layer $K_c \times 10^7$, g./sec.^{2}	2.39	4.07	4.3	4.3	4.05	4.95	4.45	4.55	5.1

Mean K_c for layers 2 to 9 is 4.6×10^{-7} g./sec.^{2}

Kieselguhr

Layer number*	1	2	3	4	5	6
W†, g.	200	300	400	500	600	700
Layer $K_c \times 10^7$, g./sec.^{2}	11.8	51.7	42.07	44.6	47	57

Mean K_c for layers 2 to 6 is 50×10^{-7} g./sec.^{2}

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*Layers built in their numerical order.
†*W* is the total mass of air dried solid in the cake.

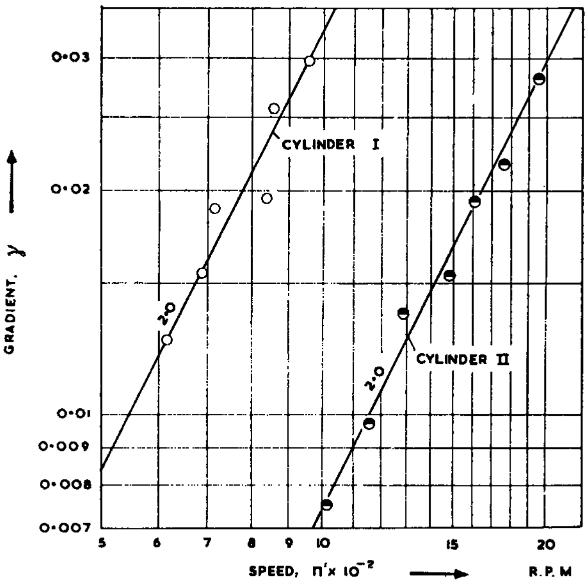

Fig. 4. The dependence of gradient γ on speed of revolution n agrees with the expected power 2.0 from Equation (10) (*15*).

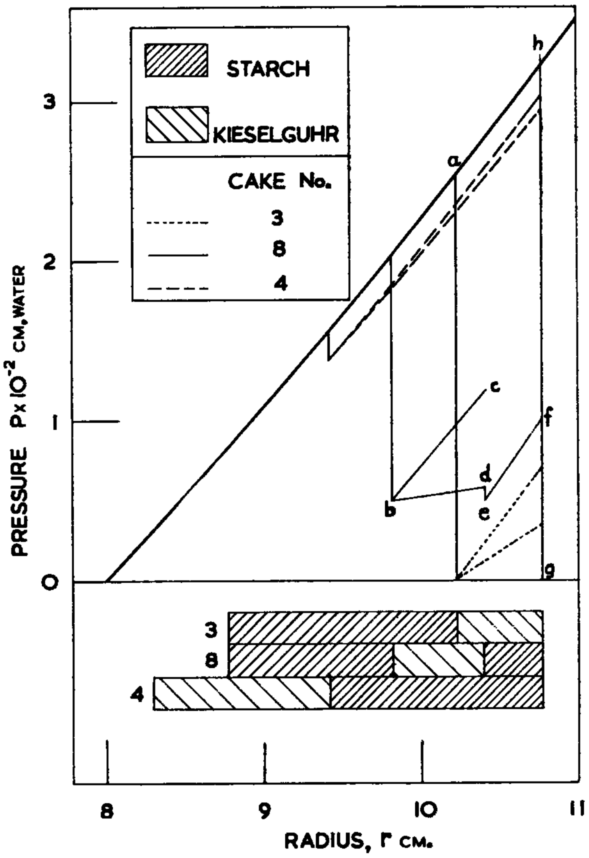

Fig. 5. Pressure distribution with banded cakes in the 9-in.-diam. hydroextractor, with n^1 = 1,080 rev./min. and r_L — 8.0 cm. (*11*).

$$q_s = \frac{CX}{\ln r_0/r_c} \tag{19}$$

where

$$C = \frac{4\pi^3 n^2 K_c(r_0^2 - r_L^2)}{\mu g\left(1 - \frac{s}{\sigma}\right)}$$

Thus

$$\frac{dV}{dt} = \frac{CX}{\ln\left[\frac{r_0}{(r_0^2 - C_1 V)^{1/2}}\right]} \tag{20}$$

and integrating from $V = 0$ at $t = 0$ to $V = V$ at $t = t$ with C and C_1 assumed independent of time gives

$$(1 + 2 \ln r_0) V + \left(\frac{r_0^2}{C_1} - V\right)$$

$$\cdot \ln (r_0^2 - C_1 V) - \frac{2r_0^2}{C_1} \cdot \ln r_0$$

$$= 2CXt \tag{21}$$

This formation method is that depicted in Figure 1*a*, ref. *14*; however, the cake may form in a very different manner. Figure 3 in ref. *14* shows contour measurements made at stages in a cake's growth and represents many such assessments using the probe and layer-dyeing techniques. It is noteworthy that although dilute slurry fed to the bottom of the cake is spread out along the vertical height X very rapidly, the solid sedimentation must also be very rapid to lead to the forms shown. The limitation caused by r_L is also clear. The formation mechanism seems independent of the presence of cloth only as the original backing, as in Figure 3 of ref. *14* or cloth with a cylindrical cake as in Figure 3c. The extreme case of such growth is shown in Figure 1 (*14*), wherein the cake is pictured as growing vertically along an x axis while confined between the radii r_1 and r_2. For this case

$$\frac{sV}{\sigma_p} = \pi x(r_1^2 - r_2^2) \tag{22}$$

or $x = C_2 V$, where $C_2 = s/\pi\sigma_p(r_1^2 - r_2^2)$ and $r_2 < r_1$. Assuming that the cake is being formed on an existing cylindrical cake of inner radius r_1, outer radius r_0, and permeability K_c results in

$$q_s = \frac{4\pi^3 n^2 K_c(r_0^2 - r_L^2)}{\mu g\left(1 - \frac{s}{\sigma}\right)}$$

$$\cdot\left[\frac{x}{\ln r_0/r_2} + \frac{X - x}{\ln r_0/r_1}\right] \tag{23}$$

whence

$$\frac{dV}{dt} = C\left[\frac{C_2 V}{\ln r_0/r_2} + \frac{X - C_2 V}{\ln r_0/r_1}\right] \tag{24}$$

$$= R_1 V + R_2 \tag{25}$$

where

$$R_1 = CC_2\left[\frac{1}{\ln r_0/r_2} - \frac{1}{\ln r_0/r_1}\right]$$

$$R_2 = \frac{CX}{\ln r_0/r_1}$$

Integrating from $V = 0$ at $t = 0$ to $V = V$ at $t = t$ and assuming R_1 and R_2 independent of time gives

$$t = \frac{1}{R_1} \cdot \ln\left[1 + \frac{R_1}{R_2} \cdot V\right] \tag{26}$$

The equation has significance only for $0 < x < X$.

Results for measured starch slurry rates q_s to maintain a constant r_L are shown in Figure 6 and also in Figure 4 of ref. *14*. The permeability of the cake materials, initial cake geometry, and slurry characteristics are all known (ref. *14*, Table 1) and one may calculate the curves A, A^1 in Figure 4 (*14*) based on Equation (21) for cylindrical growth and curves B, B^1 based on Equation (26). The experimental data show a good agreement with the vertical growth curves. The set A, B is for growth on an existing cylindrical starch cake and set A^1, B^1 for growth on cloth. To provide a comparison Equations (21) and (26) were based on an initial cake thickness of 1 mm. Further data in Figure 6 show similarly good agreement with Equation (26). It is clear that it cannot be assumed that cakes always grow cylindrically. In each case depending on material and feed method it may be necessary to check cake contours during growth and produce an appropriate equation for a simplified model of the growth mechanism.

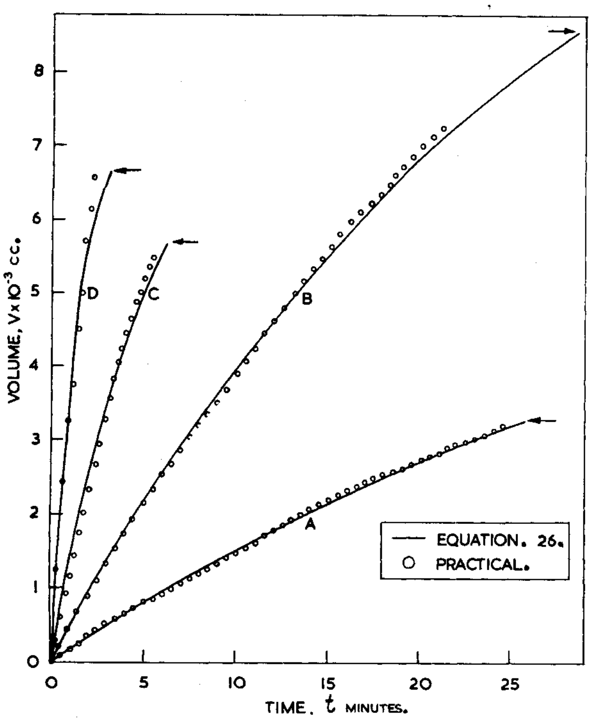

Fig. 6. Agreement between practical results and Equation (26). The arrow indicates the volume of slurry equivalent to the cake volume to fill the basket (*14*).

DISCUSSION

This review has shown that the author's proposals as to flow mechanism and permissible assumptions in submerged cakes were correct. The close agreement between filtration and hydroextraction permeabilities is considered sound proof that the flow in hydroextractor cakes has no major peculiarity. The many analytical and testing methods have also provided various techniques for use at all levels of industrial "accuracy," from those suitable for application to research apparatus to those which can be applied to large-scale operating equipment. It must be emphasized that the work reported here covers only the first problem of centrifugation. The equations so far derived are all dependent on the pressure distributions in a cake wherein the pores run full of liquid. This requires that at all times the cake is submerged, the liquid surface r_L never exceeding r_c and the liquid filling the cake to the outer radius r_0.

There are many problems of liquor flow and liquor retention, the analyses of which require other fundamental relationships. Cake formation methods are used which do not conform to the submerged cake mechanism, e.g., where the cake is formed by spraying a thick slurry or adding a crystal "mush" to the basket. The significant basic relationships are those associated with "whizzing" to remove residual liquor from the cake. Washing the cake may be done with $r_L < r_c$, but often it will consist of spraying water onto the cake at a rate less than the critical wetting rate and the inner surface of the cake will not then run full. The flow problem may then consist of two parts, the "full-flow" state in an outer layer of the cake, and a surface flow over the particles in an inner layer of the cake. The whizzing of cakes will present similar problems of surface flow, in which the significant mean variables include the "effective thickness" of the mobile liquid films on the particle surface and the retained quantities of liquid in the cake interstices, both problems depending on centrifugal field and details of particle arrangement in the cake.

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NOTATION

A = cross-sectional area of filter bed, sq. cm.
a = cross-sectional area of falling-head tube, sq. cm.
C, C_1, C_2 = constant groups
g = gravitational acceleration, cm./sec.^{2}
H = hydrostatic head in filtration tests, cm. water
H_1, H_2 = limits of H
h = static head at axial entrance to small spinning cell, cm. water
K = permeability, (g./sec.^{2})
K_F = permeability in filtration tests, g./sec.^{2}
K_c = permeability in centrifugation, g./sec.^{2}
n = speed of revolution, rev./sec.
n^1 = speed of revolution, rev./min.
n_c = speed of revolution at critical wetting condition, rev./sec.
P_c = centrifugal pressure, cm. water
P_F = friction head, cm. water
q = drainage rate through cake, cc./sec.
q_w = drainage rate at critical wetting condition, cc./sec.
q_s = slurry flow rate, cc./sec.
R_1, R_2 = constant groups
r = radius from basket axis, cm.
r_c = radius from basket axis to inner face of cake, cm.
r_L = radius from basket axis to inner face of liquid, cm.
r_0 = radius from basket axis to outer face of cake, cm.
r_w = radius from basket axis to inner face of basket wall, cm.
s = slurry concentration, g. air-dried solid/cc. slurry
t = drainage time, sec.
t^1 = drainage time for $V_L{}^1$, sec.
V. = volume of slurry, cc.
V_c = volume of cake = $\pi X(r_0{}^2 - r_c{}^2)$ cc.
V_L = volume of liquid = $\pi X(r_c{}^2 - r_L{}^2)$, cc.
$V_L{}^1$ = volume of liquid added to basket for drainage test, cc.
W = mass of dry solid in cake, g.
X = depth of hydroextractor basket, cm.
x = height of cake, cm.
μ = viscosity of liquid, poise
γ = $n^2\lambda$
λ = $4\pi^2 K_c/\mu g \ln (r_0/r_c)$
σ = density of air-dried solid, g./cc.
σ_p = packing density of solid in cake, g./cc.

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